

# LIE–BUTCHER SERIES AND ROUGH PATHS ON HOMOGENEOUS MANIFOLDS

TWO TALKS GIVEN JOINTLY WITH  
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ABSTRACT. Butcher’s B-series is a fundamental tool in analysis of numerical integration of differential equations. In the recent years algebraic and geometric understanding of B-series has developed dramatically. The interplay between geometry, algebra and computations reveals new mathematical landscapes with remarkable properties.

The shuffle Hopf algebra, which is fundamental in Lyons’s groundbreaking work on rough paths, is based on Lie algebras without additional properties. Pre-Lie algebras and the Butcher–Connes–Kreimer Hopf algebra are providing algebraic descriptions of the geometry of Euclidean spaces. This is the foundation of B-series and was used elegantly in Gubinelli’s theory of Branched Rough Paths. Lie–Butcher theory combines Lie series with B-series in a unified algebraic structure based on post-Lie algebras and the MKW Hopf algebra, which is giving algebraic abstractions capturing the fundamental geometrical properties of Lie groups, homogeneous spaces and Klein geometries.

In these talks we will give an introduction to these new algebraic structures. Building upon the works of Lyons, Gubinelli and Hairer–Kelly, we will present a new theory for rough paths on homogeneous spaces built upon the MKW Hopf algebra. (Joint work with Ch. Curry, D. Manchon, H. Munthe-Kaas.)

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