COURSE "ALGEBRAIC AND COMBINATORIAL ASPECTS IN STOCHASTIC CALCULUS"

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In the last decade Hopf and Lie algebraic structures have reshaped the field of algebraic combinatorics. Moreover, they turn out to be crucial in the context of stochastic calculus. These developments can be traced back to G.-C. Rota's work, who uncovered beautiful links between the combinatorics of the poset of set partitions, the related Möbius calculus as well as combinatorial bialgebras, and various probabilistic and stochastic structures and phenomena, appearing, for instance, in cumulant calculus and the fine properties of stochastic integrals.

In this course we will discuss in detail shuffle and quasi-shuffle Hopf algebras together with related (pre-)Lie algebraic aspects, in the context of computations involving iterated stochastic integrals in the Stratonovich and Itô calculus framework. A particular emphasis will be put on the interplay between Hopf algebra structures (and generalizations thereof) on permutations, surjections, set partitions and rooted trees. We will extend the scope by exploring similar combinatorial and algebraic structures, which have been unveiled in the context of moment-cumulant calculus in noncommutative probability theory.

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