COURSE "FLUCTUATIONS OF CHAOTIC RANDOM VARIABLES: THEORETICAL FOUNDATIONS AND GEOMETRIC APPLICATIONS"

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In recent years, a large body of work has been devoted to the study of the asymptotic properties of those random variables that belong to the homogeneous chaos of some random measure, typically by means of variational techniques and integration by parts formulae. The aim of these lectures is to present some of the crucial aspects of the theory, by specifically focussing on combinatorial aspects, that are in particular tightly connected to the Rota–Wallstrom theory of combinatorial integration. Such a theory of integration, appearing in the fundamental paper [4] provides a unified combinatorial framework, in order to understand (multiple) stochastic integration and associated formulae as consequences of the properties of partition lattices and associated Möbius inversion formulae. Some of the topics that we will deal with in the lectures involve fourth moment theorems, asymptotic independence, Edgeworth expansions, as well as a the introduction to a recent stream of research, focussing on probabilistic approximations by means of Markov generators and carré-du-champ operators. If time permits, we will also illustrate three remarkable geometric applications: to random geometric graphs, to nodal sets of random waves, and to real polarization problems. The last topic is intimately connected to a long-standing open problem in Gaussian analysis, known as the "Gaussian product conjecture", that we will try to illustrate once again from a combinatorial standpoint.

References:

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- [2] G. Peccati, M. Reitzner (Editors) (2016), Stochastic analysis for Poisson point processes: Malliavin calculus, Wiener-Itô chaos expansions and stochastic geometry, Springer-Verlag.
- [3] G. Peccati, M.S. Taqqu (2010), Wiener Chaos: Moments, Cumulants and Diagrams, Springer-Verlag.
- [4] G.-C. Rota, T. C. Wallstrom (1997), *Stochastic integrals: a combinatorial approach*, Ann. Probab. 25, (3): 1257-1283