COURSE "COMBINATORIAL ASPECTS OF FREE PROBABILITY AND FREE STOCHASTIC CALCULUS"

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Free probability was initiated by Voiculescu in the 1980s in the context of operator algebras (in particular, the question about isomorphism of von Neumann algebras), and received a tremendous boost, when Voiculescu discovered in 1991 its relations to random matrices. Since then it has evolved into a very active subject with many relations to other fields. One particular feature of free probability is its combinatorial structure, which goes in parallel to the combinatorics in classical probability theory - one only has to replace the lattice of all partition by the lattice of non-crossing partitions.

The first lectures will introduce the basics of free probability from this combinatorial perspective. The lattice of non-crossing partitions will be used to define the notion of free cumulants. This will then allow us to deal with the additive and multiplicative free convolution.

Via a combinatorial treatment of the free central limit theorem we will then recognize the semicircular distribution as the free analogue of the Gaussian distribution. This will be pushed further to the definition of the free analogue of the Brownian motion. We will see free analogues of Ito's formulas and also of Malliavin calculus; again an emphasis will be on a combinatorial understanding of the difference between the classical and the free situation.

Finally, we will also address some recent rough path approaches to free, and more generally non-commutative, stochastic integrations.

References:

- Roland Speicher (2017),
  Free probability theory and its atavars in representation theory, random matrices, and operator algebras; also featuring: non-commutative distributions, Jahresber. Dtsch. Math. Ver 119 (2017), 3-30.
- Alexandru Nica, Roland Speicher (2006),
  Lectures on the Combinatorics of Free Probability, London Mathematical Society Lecture Notes, vol. 335, Cambridge University Press, 2006.
- Philippe Biane, Roland Speicher (1998),
  Stochastic calculus with respect to free Brownian motion and analysis on Wigner space, Prob. Th. Rel. Fields 112 (1998), 373-410
- Todd Kemp, Ivan Nourdin, Giovanni Peccati, Roland Speicher (2012),
  Wigner chaos and the fourth moment, Ann. Probab. 40 (2012), 1577-1635.
- Aurélien Deya, René Schott (2013)
  On the rough-paths approach to non-commutative stochastic calculus, J. Funct. Anal. 265 (2013): 594-628.